Bahrain Mental Math Olympiad 2025

Referral Notes for Grand Master Category (Age : 12 years and above)

Syllabus: -

- Mental Addition
- Mental Subtraction
- Exponents
- Prime Factorization
- Profit & Loss
- Mental Multiplication
- Mental Division
- Simple Ratios
- Average/ Mean
- HCF & LCM
- Squaring Numbers
- Cubing Numbers
- Cube Roots
- Square Roots
- Percentage

Participants have to prepare Round 1 to Round 7 on the APP as per their Age Category

To gain a better understanding of the types of problems and their difficulty levels, please refer to the Practice Sheets and Sample Question Banks provided. They offer a comprehensive overview of the various types of problems and the degree of complexity involved.

NOTE - Please note that the referral material provided is intended only as a reference. If your child is currently following a specific method, we recommend that you continue with that approach. These resources are intended to support children who may be unfamiliar with certain mental math topics and are seeking additional assistance.

MATH COMPETITIONS AND LEAGUE

Mental Additions Methods

Sometimes, we find it difficult to add numbers which end in 6, 7, 8 and 9. For example, if we have 16+9, that's a difficult problem to do mentally.

But we can make it easy. We can use a method called by addition and by subtraction'.

Method 1

Let us try 16 + 9.

Since adding 9 directly is difficult for most of us, we add 10 which is easy to do-mentally. So, since 9 is 1 less than 10, we can add 10 and then subtract 1 from our answer. Our sum looks like this:

We do:

1) 16+ 10 = 26
2) 26 -1= 25 is our answer.

Here, we would like to draw your attention to the method which is called 'by addition and by subtraction'. So, we add first and then subtract.

Let's take another example. Say, we have:

68 +9

We do

68 + 10 = 78.

78 - 1 = 77 our answer.

Now, let's try another example on our own.

59 + 8

We do: 59 + 10 = 69

Since 8 is 2 less than 10, we do:

69- 2 = 67.

The answer is 67.

Now can you tell us what happens if one of the numbers Method

being added was closer to 20?

Say, we have 116 + 18

So, to add 18, we add 20 and then subtract the 2.

So, we do:

116 + 20 = 136

136 - 2 = 134. This is our answer.

Now let's try addition with a larger number 139 +69

Here, to add 69, we add 70 and then subtract 1 from the total

139 + 70 = 209

209 - 1 = 208 our answer.

Let's try another problem.

166 +88

To add 88, we added 90 and then subtract 2.

166 +90 = 256

256 -2=254 was the answer.

ACTIVITY 1

Find the Sum									
1	27	2	59	3	29	4	57	5	19
	+5		+6		+9		+7		+7
Ans		Ans		Ans		Ans		Ans	
6	229	7	964	8	315	9	428	10	886
	+88		+96		+99		+89		+85
Ans		Ans		Ans		Ans		Ans	

Method 2

Left to Right Mental Addition

This method is called Left to Right Mental Addition Method. So far traditionally in Maths, we have been doing additions and other operations from Right to Left.

However, we can also do mental addition effectively from Left to Right.

Say for example we have

$$\begin{array}{r} 88 \\ \underline{+\ 34} \end{array}$$

Step 1, we first add the figures in the left column.

So 8 + 3 = 11(We keep this figure in our head)

$$\begin{array}{l} \ \ 88 \\ \underline{+\ 34} \\ 11, \end{array}$$

Step 2, we now add the figures in the Right Hand Column

8 + 4 = 12(we keep this in our head too)

The sum now looks like this:

$$\begin{array}{l} \ \ 88 \\ \underline{+\ 34} \\ 11,12 \end{array}$$

Step 3, in the final step we add the middle digits

$$\begin{array}{l} \ \ 88 \\ \underline{+\ 34} \\ 11,12 \end{array}$$

122

So, the answer is 122

In our next example let's try adding two 3 digit numbers

Add 482 + 859

482
+859

Step 1:
Start by adding the column left to right.
The first column is 4 +8 = 12
The Middle Column is 8 + 5 = 13

482
+859
12,13

Step 2:
We add the middle digit of the first 2 columns.
So we have 133 in our head.
Then we add the last column on the right
So we have 2 + 9 = 11

482
+859
133,11

Step 3:
So, in our mind we have 133,11
We then add the digits on either side of the comma.
In this case, we add 3 and 1 and we get 4.
So our final answer is 1341

482
+859
1341

Find the Sum									
1	47	2	85	3	47	4	64	5	245
	+88		+16		+29		+47		+257
Ans		Ans		Ans		Ans		Ans	
6	239	7	864	8	818	9	2,428	10	5,886
	+188		+596		+399		+1,899		+2,285
Ans		Ans		Ans		Ans		Ans	

Mental Subtractions

Sometimes it becomes difficult to subtract numbers like 6, 7, 8, 9 from numbers ending in 1, 2, 3, 4, 5. This new method provides us with a different view and simpler approach, just like we saw in addition.

Say, we have 42 -9

We subtract 10 first (as it is easy to take away 10) and then add back 1 (since 9 is 1 less than 10).

42 - 10 = 32

So, 32 + 1 = 33. This is the answer.

Let us take another example:

81 - 8

So, we subtract 10 first (as it is easy to add back 2 (since 8 is 2 less than 10).

81 - 10 = 71

So, 71 + 2 = 73 is the answer

Now let's try one with a slight variation!

43- 17

So, we subtract 20 first (as it is easy to take away 20) and then add back 3 (since 17 is 3 less than 20).

43 - 20 = 23

So, 23 + 3 = 26 is the answer.

Let's try another variation 272 - 28

Here, we will first subtract 30 and then add 2.

272 - 30 = 242

So, 242 + 2 = 244 is the answer.

Find the Difference									
1	77	2	56	3	28	4	54	5	17
	-9		-7		-9		-7		-9
Ans		Ans		Ans		Ans		Ans	
6	122	7	464	8	315	9	826	10	886
	-69		-96		-49		-87		-88
Ans		Ans		Ans		Ans		Ans	

Introduction to Exponents

The exponent of a number says how many times to use that number in a multiplication.

It is written as a small number to the right and above the base number. Exponential expression has two components i.e. Base and Exponent.

Other names for exponent are index or power

8^2 → Exponent (Power/Index)

↓

Base

Here's another example: the base is 4, and the exponent is 3

4^3

An exponent tells us to multiply the base by itself that number of times. In our example, we will multiply the base of 4 by itself 3 times:

$4^3 = 4x4x4$

Once we write out the multiplication problem, we can easily evaluate the expression. Let's do this for the example we've been working with

$4^3 = 4x4x4$

$= 16 \times 4$

$= 64$

Another example: $9^2 = 9 \times 9 = 81$
(The exponent "2" says to use the 9 two times in a multiplication.)

Another example: $5^3 = 5 \times 5 \times 5 = 125$
(The exponent "3" says to use the 5 three times in a multiplication.)

Prime Factorization

Some numbers *only have two divisors*: 1 and the number itself. Such numbers are called **prime numbers.** 11 is one of them.

factor		factor		product
1	**×**	**11**	**=**	**11**

The prime numbers less than 30 are **2, 3, 5, 7, 11, 13, 17, 19, 23, and 29**. One is usually not counted as a prime number.

Prime factorization using a factor tree

A factor tree is a handy way to factor numbers to their prime factors. The factor tree starts at the root and grows upside down!

We want to factor 24 so we write 24 on top. First, 24 is factored into 4 × 6. However, 4 and 6 are not primes, so we can *continue* factoring. Four is factored into 2 × 2 and six is factored into 2 × 3.

We will not factor 2 or 3 any further because they are prime numbers.

```
(root)          24
               /  \
              4  ×  6
             / \   / \
(leaves)    2 × 2 × 2 × 3
```

Once you get to the primes in your "tree", they are the "leaves", and you stop factoring in that "branch". So 24 = 2 × 2 × 2 × 3. This is the ***prime factorization of 24.***

Examples:

```
30
/ \
5 × 6
   / \
  2 × 3
```

5 is a prime number—it is a "leaf". Once done, "pick the leaves"—you can even circle them to see them better! So, 30 = 2 × 3 × 5.

```
 21
 / \
3 × 7
```

Both 3 and 7 are prime numbers, so we cannot factor them any further. So 21 = 3 × 7.

```
   66              66
  /  \            /  \
11 × 6    OR     2 × 33
    / \              / \
   2 × 3           11 × 3
```

You can start the factoring process any way you want. The end result is the same: 66 = 2 × 3 × 11.

Prime numbers are like building blocks of all numbers. They are the first and foremost, and other numbers are "built" from them. "Building numbers" is like factoring backwards. We start with the building blocks—the primes—and see what number we get:

```
2 × 5 × 2 × 2
 \ /     \ /
 10   ×   4
   \     /
     40
```

```
2 × 3 × 2 × 3 × 2
 \ /     \ /    |
  6   ×   6   × 2
  |        \    /
  6   ×      12
    \       /
        72
```

```
5 × 2 × 7
 \ /    |
 10   × 7
   \   /
    70
```

```
2 × 7 × 2×3
 \ /    \ /
 14   ×  6
   \    /
    84
```

By using the process above (building numbers starting from primes) you can build ANY whole number there is! Can you believe that?

We can say this in another way: **ALL numbers** can be factored so the factors are prime numbers. That is sort of amazing! This fact is known as the *fundamental theorem of arithmetic*. Indeed, it is fundamental.

```
          992
         /    \
     4    ×   248
    / \       /   \
  2 × 2  ×  4 × 62
           / \   / \
          2 × 2 × 2 × 31
```

So, no matter what the number is—992 or 83,283 or 150,282—it can be written as a product of primes.

See 992 factored on the right. 992 = 2 × 2 × 2 × 2 × 2 × 31. For 83,283 we get 3 × 17 × 23 × 71, and 151,282 = 2 × 3 × 3 × 3 × 11 × 11 × 23.

To find these factorizations, you need to test-divide the numbers by various primes so it is a bit tedious. Of course, computers can do the divisions very quickly

Profit and Loss

Cost Price: The amount paid to purchase an article or the price at which an article is made, is known as its cost price. The cost price is abbreviated as C. P.

Selling Price: The price at which article is sold, is known as its selling price. The selling price is abbreviated as S. P

Profit: If the selling price (S.P.) of an article is greater than the cost price (C.P), then the difference between the selling price and cost price is called profit.
Thus, If S. P. > C.P., then

Profit = S. P. – C. P.
⇒ S. P. = C. P. + Profit
⇒ C. P. = S. P. – Profit.

Example:
An article was brought for Rs 75 and sold for Rs 95 Find the gain or loss.
Solution:

CP of the article = 75
SP of the article = 95
Since SP > CP, so there is a profit
Profit = SP – CP
Profit = 95- 75 = 20
So Profit is Rs 20

Loss: If the selling price (S.P.) of an article is less than the cost price (C.P.), then the difference between the cost price (C.P.) and the selling price (S.P.) is called loss.
Thus if S.P. < C.P., then

Loss – C.P. – S.P.
⇒ C. P. = S. P. + Loss
⇒ S. P. = C. P. – Loss

Example:
An article was brought for Rs 50 and sold for Rs 40 Find the gain or loss.
Solution:

CP of the article = 50
SP of the article = 40
Since SP < CP, so there is a Loss
Loss = CP – SP
Loss = 50 - 40 = 10
So Loss is Rs 10

Multiplication

To do Mental Multiplications of 2 digit numbers and higher, one needs to know tables of 2-9 by heart

Any multiplication has two parts **multiplicand** and **multiplier**. The multiplicand is the number to which we multiply and multiplies is the numbers by which we multiply.

Example: Multiply 76 by 84.

Here 76 is a multiplicand and 84 is a multiplier.

Example: 986 x 487

Here 986 is a multiplicand and 487 is a multiplier.

Also note then in multiplication.

The computation made by multiplier to digits of multiplicand is from right to left.

e.g., in multiplication 164 by 5.

164 is multiplicand and 5 is multiplier. In multiplication we consider 4 first, then 6 and afterward 1 (from right to left).

Multiplication of Two Digit Multiplicand by Two Digit Multiplier

Step 1: The right hand digit of multiplicand is multiplied by right hand digit of multiplier.

Step 2: Apply inside-outside principle (plus carry).

Step 3: The left hand digit of multiplicand is multiplied by left hand digit of multiplier (plus carry).

Example: Multiply 46 by 27.

46 x 27

Step 1: Right hand digit 6 of multiplicand 46 is multiplied by right hand digit 7 of multiplier 27

6x7 = 42; write 2. Carry over 4

46 x 27
2

Step 2: Apply inside-out principle [(6x 2) + (4 x 7) =12+ 28 = 40 Plus carry over. (4)

40 + 4 = 44; write 4 and carry over 4

46 x 27
42

Step 3: Left hand digit 4 of multiplicand 46 is multiplied by left hand digit 2 of multiplier 27

4 x 2 = 8

Plus carry of the previous step. (4)

8 + 4 = 12; write 12

46 x 27
1242

The answer is 1242

Exercise

1. Multiply 43 by 36.
2. Multiply 78 by 31.
3. Multiply 98 by 77.

Multiplication by Two Digit Multiplier

Step 1: The right most digit of multiplicand is multiplied by right most digit of multiplier.

Step 2: For each successive pair of digits of multiplicand and multiplier apply inside-outside principle (plus carry)

Step 3: The left most digit of multiplicand is multiplied by left most digit of multiplier (plus carry).

Example: Multiply 987432 by 56. 987432 x 56

Step 1: Right most digit 2 of multiplicand is multiplied by right most digit 6 of multiplier

2 x 6 = 12. write 2. carry over 1

987432 x 56
2

Step 2: For **pair 32 and multiplier 56**.

apply inside-outside principle (plus carry 1)

(2 x 5) + (3 x 6)] + 1

= 10 + 18 + 1 = 29:

write 9, carry over 2

987432 x 56
92

For **pair 43 and multiplier 56**

apply inside-outside principle (plus carry 2)

(3 x 5) + (4 x 6)| + 2

= 15 + 24 + 2 = 41:

write I, carry over 4

987432 x 56
92

For **pair 74 and multiplier 56**

apply inside-outside principle + (carry 4)

(4 x 5) + (7 x 6) + 4

= 20 + 42 + 4 = 66:

write 6, carry over 6

987432 x 56
6192

For **pair 87 and multiplier 56**

Apply inside-outside principle + carry

(7 x 5) + (8 x 6)] + 6 (carry)

= 35 + 48 +6 = 89:

write 9. carry over 8

987432 x 56
96192

For **pair 98 and multiplier 56**

apply inside outside principle + carry

[(8 x 5) + (9 x 6) + 8 (carry)

= 40 + 54 + 8 = 102.

write 2. carry over 10

987432 x 56
296192

Step 3: The left most digit 9 of multiplicand

is multiplied by left most digit 5 of

multiplier (plus carry 10)

(9 x 5) + 10 = 45 + 10 = 55;

write 55

987432 x 56
55296192

The answer is 55296192

Exercise

1. Multiply 99641 by 76.
2. Multiply 12011 by 33
3. Multiply 96656 by 77

Multiplication by Three Digit Multiplier

Step 1: The right most digit of multiplicand is multiplied by right most digit of multiplier.

Step 2: For the right most pair of multiplicand and right most pair of multiplier apply inside-outside principle (plus carry)

Step 3: For each successive triplet of multiplicand and multiplier apply inside-outside principle (plus carry).

Step 4: For the left most pair of multiplicand and left most pair of multiplicand apply inside-outside principle (plus carry)

Step 5: The left most digit of multiplicand is multiplied by left most digit of multiplier (plus carry),

Example: Multiply 9870321 by 654.

<u>9870321 x 654</u>

Step 1:

Multiply 1 by 4:

1x4 = 4;

write 4

<u>9870321 x 654</u>
4

Step 2: For pair 21 and pair 54 apply

inside outside principle;

(1 x 5) + (2 x 4)] = 5 + 8 = 13;

write 3, carry over 1

<u>9870321 x 654</u>
34

Step 3:

For triplet (321) and multiplies (654)

apply inside-outside principle (+ carry 1)

[(1 x 6) + (2 x 5) + (3 x 4)] + 1

= 6 + 10 + 12 + 1 = 29;

write 9, carry over 2

9870321 x 654
934

- For triplet (032) and multiplier (654)

Apply inside-outside principle (+ carry 2)

(2 x 6) + (3 x 5) + (0 x 4)] + 2 (carry)

= 12 + 15 +0 + 2 = 29.

write 9, carry over 2

9870321 x 654
9934

For triplet (703) and multiplier 654

apply inside-outside principle (+ carry 2)

[(3 x 6) + (0 × 5) + (7x4)] +2 (carry)

= 18 + 0 + 28 + 2 = 48;

write 8, carry over 4

9870321 x 654
89934

For triplet (870) and multiplier 654

apply inside-outside principle (+ carry 4)

[(0 x 6) + (7 x 5) + (8 x 4)] + 4 (carry)

= 0 + 35 + 32 + 4 = 71;

write 1, carry over 7

9870321 x 654
189934

For triplet (987) and multiplier 654

apply inside-outside principle (+ carry 7)

|(7x 6) + (8 x 5) + (9 x 4)] +7 (carry)

= 42 + 40 + 36 + 7 = 125,

write 5, carry over 12

9870321 x 654
5189934

Step 4: For pair (98) and pair (65)

apply inside-outside principle (+ carry 12)

[(8 x 6) + (9 x 5)] + 12 (carry)

= 48 + 45 + 12 = 105:

write 5, carry over 10.

9870321 x 654
55189934

Step 5: Multiply 9 by 6 (plus carry 10)

(9 x 6) + 10 = 54 + 10 = 64;

write 64

9870321 x 654
6455189934

The answer is 6455189934

Exercise

1. Multiply 25287 by 999.
2. Multiply 30987 by 684.
3. Multiply 23557 by 786.

Multiplication by Four Digit Multiplier is multiplied

Step 1: The right most digit of multiplicand is multiplied by right most digit of multiplier.

Step 2: For the right most pair of multiplicand and right most pair of multiplier apply inside-outside principle (plus carry)

Step 3: For the right most triplet of multiplicand and right most triplet of multiplier apply inside-outside principle (plus carry)

Step 4: For each successive quadruple of multiplicand and multiplier apply inside-outside principle (plus carry)

Step 5: For the left most triplet of multiplicand and left most triplet of multiplier apply inside-outside principle (plus carry).

Step 6: For the left most pair of multiplicand and left most pair of multiplier apply-inside-outside principle (plus carry).

Step 7: For left most digit of multiplicand is multiplied by leftmost digit of multiplier (plus carry),

Example:

Multiply 987021 by 6543 — 987021 x 6543

Step 1:

Multiply 1 by 3:

Ix 3 = 03,

write 3 — 987021 x 6543
3

Step 2:

For pair (21) and (43)

apply inside-outside principle:

[(1 x 4) + (2 x 3)

= 4 + 6 = 10,

write 0, carry over 1 — 987021 x 6543
03

Step 3:

For triplet (0 2 1) and (543) apply

inside-outside principle; plus carry 1

(I x 5) + (2 x 4) + (0 x 3) + 1

= 5 + 8 + 0 + 1 = 14

write 4, carry over 1

987021 x 6543
403

Step 4:

For quadruple (7021) and multiplier

apply inside outside principles

(6343)+1 (carry)

[(1 x 6) + (2x 5) + (0 x 4) + (7x 3)] + 1

6+10+0+21 + 1 = 38,

write 8, carry over 3

987021 x 6543
8403

For quadruple (8702) and multiplier

apply inside-outside principle;

(6543) + 3 (carry);

[(2x 6) + (0 x 5) + (7 x 4) + (8 x 3)] + 3

= 12 +0+ 28 + 24 + 3 = 67:

write 7. carry over 6

987021 x 6543
78403

For quadruple (9870) and multiplier

apply inside-outside principle:

(6543) + 6 (carry):

[(0 x 6) + (7x 5) + (8 x 4) + (9x 3)| +6

= 0 + 35 + 32 +27 +6 = 100;

write 0, carry over 10

987021 x 6543
078403

Step 5:

Triplet (987) and (654) apply inside-outside principle;+ 10 (carry);

[(7 x 6) + (8 x 5) + (9 x 4)] + 10

= 42 +40 + 36 + 10 = 128;

write 8, carry over 12

987021 x 6543
8078403

Step 6:

Pair (98) and (65) apply

inside outside principle + 12 (carry);

[(8 x 6) + (9x 5)] + 12

= 48 + 45 + 12 = 105:

write 5, carry over 10

987021 x 6543
58078403

Step 7:

Multiply 9 by 6:

9x6=54 + 10 (carry) = 64;

write 64

987021 x 6543
6458078403

The answer is 6458078403

Exercise

I. Multiply 221310 by 2524.

2. Multiply 1234567 by 8585.

3. Multiply 256879 by 7676

Inside-outside principle: Consider the multiplicand and multiplier with same number of digits

- Multiply successive right hand digit of multiplicand by successive left hand digit of multiplier.
- Add the product

Multiplication	Steps
Of two digit multiplicand by two digit multiplier	• Right hand digit of multiplicand is multi plied by right hand digit of multiplier. I Apply principle (plus carry) • Apply inside-outside principle (plus carry) • Left hand digit of multiplicand is multiplied by left hand digit of multiplier(plus carry)
With two digit multiplier	• Right hand digit of multiplicand us multiplied by right hand digit of multiplier. • Successive pair of digits of multiplicand and multiplier apply inside-outside principle(plus carry) • Left hand digit of multiplicand is multiplied by left hand digit of multiplier(plus carry)
With three digit multiplier	• Right most digit of multiplicand is multiplied by right hand digit of multiplier. • Right most pairs of multiplicand and right most pair of multiplier-apply inside outside principle (+ carry). • Successive triplets of multiplicand and multiplier apply inside-outside principle (+ carry). • Left most pair of multiplicand and left most pair of multiplier apply inside outside principle (+ carry). • Left hand digit of multiplicand is multiplied by left hand digit of multiplier(+ carry).
With four digit multiplier	• Right most digit of multiplicand is multi plied by right hand digit of multiplier. • Right most pair of multiplicand and right most pair of multiplier-apply inside outside principle (+ carry). • Right most triplet of multiplicand and right most triplet of multiplier apply inside-outside principle (+ carry). • Successive quadruplet of multiplicand and multiplier apply inside-outside principle (+ carry). • Left most triplet of multiplicand and left most triplet of multiplier-apply inside outside principle (+ carry). • Left most pair of multiplicand and left most pair of multiplier apply inside outside principle (+ carry). • Left hand digit of multiplicand is multiplied by left hand digit of multiplier (+ carry).

Division

Dealing with multiplication of fairly considerable length, we now move to division in comparison to multiplication is considered tough, yet the truth is that division is as simple as multiplication and faster

Naming the Parts of a Division

A division has four parts which are called **divisor, dividend, quotient and remainder**. The following rhyme will help you to remember the parts of division

"The divisor is the number that divides the dividend, the answer is the quotient, the remainders at the end."

Example : Divide 2862 by 4.

2862÷4 = 715+2

In the example the divisor is 4. because it divides the dividend 2862. The quotient is 715 (which is the result of division) and the remainder is 2 (because it is that which remains)

Consider this also

The divisor is the number that divides.

The dividend is the number that is being divided.

The quotient is the result of division

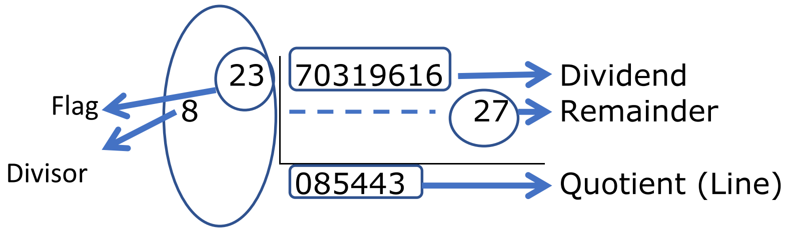
The remainder is that remains after division

and also the remainder is always less than the divisor.

Example: Divide 70319616 by 823. Here 70319616 is dividend and 823 is divisor. On dividing

70319616 (dividend) by 823 (divisor) one gets 85443 as quotient and 27 as remainder. This can be written as

70319616 ÷ 823 = 85443 remainder 27



Above we have written divisor 823 as 823 where 23 is called a flag. Note here 23 is not the power of 8. We will see later in this chapter the use of flag.

In respective of multiplication in division we make computations dividend from left to right.

In division we make computation with dividend from left to right

e.g., in dividing 123 by 6.

Here 123 is dividend and 6 is divisor.

In dividend we consider I first, then 2 and afterward 3 (from left I to right digit 3).

Division by Single Digit Divisor

Divide remainder and successive digit (from left or right) divisor

Example: Divide 8396 by 6.

$$\begin{array}{r|l} 6 & 8\ 3\ 9\ 6 \\ \hline & \end{array}$$

Divide remainder and successive digit (from left to right) by divisor.

- Since there is no remainder, divide 8 by
  divisor 6;
  8÷6=1 remainder 2.

$$\begin{array}{r|l} 6 & 8\ 3\ 9\ 6 \\ & \ \ 2 \\ \hline & 1 \end{array}$$

- Divide 23 (remainder 2, successive digit 3) by divisor 6;
  23 ÷ 6 = 3 remainder 5.

$$\begin{array}{r|l} 6 & 8\ 3\ 9\ 6 \\ & \ \ 2\ 5 \\ \hline & 1\ \ 3 \end{array}$$

- Divide 59 (remainder 5, successive digit 9) by divisor 6
-

  59÷6 = 9 remainder 5.

$$\begin{array}{r|l} 6 & 8\ 3\ 9\ 6 \\ & \ \ 2\ 5\ 5 \\ \hline & 1\ \ 3\ \ \ 9 \end{array}$$

- Divide 56 (remainder 5. Successive digit 6) by divisor 6;
-

  56 ÷ 6 = 9 remainder 2

$$\begin{array}{r|l} 6 & 8\ 3\ 9\ 6 \\ & \ \ 2\ 5\ 5\ \ 2 \\ \hline & 1\ \ 3\ \ \ 9\ 9 \end{array}$$

The answer is 1399 remainder 2.

Exercise

Divide, leaving whole number remainders

1. Divide 454 by 3	Ans. 151 rem. 1
2. Divide 4112 by 3	Ans: 1370 rem. 2
3. Divide 6599 by 6	Ans. 1099 rem. 5
4. Divide 7455 by 6	Ans. 1242 rem. 3

LIVE MATH COMPETITIONS AND LEAGUE

Division by Two Digit Divisor

Step 1: Divide first digit of dividend by first digit of divisor Check: (Remainder and successive digit of dividend - (Flag x Quotient

= Negative numbers choose quotient that make this positive

Step 2: Divide it by first digit of divisor.

Check: (Remainder and successive digit of dividend) - (Flag x Quotient)

=Negative numbers choose quotient that make this positive

Step 3: (Remainder and last digit of dividend) - (Flag x Quotient) = Remainder of division

Example: Divide 9137 by 43

$$\begin{array}{r|llll} 3 & 9 & 1 & 3 & 7 \\ 4 & & & & \\ \hline \end{array}$$

Step 1: 9 ÷ 4 = 2 remainder 1.

Check: 11- (3 x 2) = 11-6

= 5

$$\begin{array}{r|llll} 3 & 9 & 1 & 3 & 7 \\ 4 & & 1 & & \\ \hline & 2 & & & \end{array}$$

Step 2:

- 11- (3 x 2) = 11 - 6 = 5:
- 5 ÷ 4 = 1 remainder 1.
  Check: 13- (3 x 1) = 13-3= 10

$$\begin{array}{r|llll} 3 & 9 & 1 & 3 & 7 \\ 4 & & 1 & 1 & \\ \hline & 2 & & & \end{array}$$

- 13-(3x1) = 13-3 = 10;
  10 ÷ 4 = 2 remainder 2.
  Check: 27-(3 x 2) = 27 - 6 = 21

$$\begin{array}{r|llll} 3 & 9 & 1 & 3 & 7 \\ 4 & & 1 & 1 & 2 \\ \hline & 2 & 1 & 2 & \end{array}$$

Step 3: 27- (3 x 2) = 27 - 6 = 21.

$$\begin{array}{r|lllll} 3 & 9 & 1 & 3 & 7 & \\ 4 & & 1 & 1 & 2 & 21 \\ \hline & 2 & 1 & 2 & & \end{array}$$

The answer is 212 remainder 21.

```
3 | 3 8 9 8 2
7 |__________
```

Example: Divide 38982 by 73

Step 1: 3 ÷ 7 = 0 remainder 3.

Check: 38- (3 x 0) = 38-0

= 38

```
3 | 3 8 9 8 2
7 |   3
  |__________
    0
```

Step 2:

- 38- (3 x 0) = 38 - 6 = 38:
  38 ÷ 7 = 5 remainder 3.
  Check : 39 - (3 x 5) = 39-15= 24

```
3 | 3 8 9 8 2
7 |   3
  |__________
    0
```

- 39-(3x5) = 39-15 = 24;
  24 ÷ 7 = 3 remainder 3.
  Check: 38-(3 x 3) = 38 - 9 = 29

```
3 | 3 8 9 8 2
7 |   3 3
  |__________
    0 5
```

- 38 – (3 x 3) = 38 – 9 = 29;
  29 ÷ 7 = 4 remainder 1.
  Check: 12-(3 x 4) = 12 – 12 =0

```
3 | 3 8 9 8 2
7 |   3 3 3
  |__________
    0 5 3
```

Step 3: 12- (3 x 4) = 12 - 12 = 0.

```
3 | 3 8 9 8 2
7 |   3 3 3 1 0
  |____________
    0 5 3 4
```

The answer is 534 remainder 0.

Exercise (Find Quotient and Remainder)

1. Divide 7543 by 43
2. Divide 5534 by 86
3. Divide 8188 by 94
4. Divide 4524 by 51
5. Divide 8786 by 86

Simplifying Ratios

Many ratios can be written with smaller numbers. This is called writing ratios in their simplest form, or simplifying ratios.

Simplifying ratios makes them easier to work with.

Note: To simplify ratios you can use the same technique that is used to simplify fractions.
As the two examples below show, you simplify ratios by dividing the number on each side by their greatest common factor.

Simplify 15 : 9

Ratio	15 : 9
Factors of 15	1 , 3 , 5 , 15
Factors of 9	1 , 3 , 9
Greatest Common Factor (G.C.F)	3
Divide both by G.C.F	15 ÷ 3 = 5 9 ÷ 3 = 3
Ratio in simplest form	5 : 3

Simplify 6 : 30

Ratio	6 : 30
Factors of 6	1,2,3,6
Factors of 30	1,2,3,5,6,10,15
Greatest Common Factor (G.C.F)	6
Divide both by G.C.F	6 ÷ 6 = 1 30 ÷ 6 = 5
Ratio in simplest form	1 : 5

Average & Mean

The formula for calculating Average and mean are the same,

The most widely used method of calculating an average is the 'mean'.

Mean is a point in a data set which is the average of all the data point we have in a set. It is basically arithmetic average of the data set and can be calculated by taking a sum of all the data points and then dividing it by the number of data points we have in data set.

$$\text{Mean} = \frac{\text{Sum of all Data Points}}{\text{Number of Data Points}}$$

Example
1) Find Mean of 25,20,15,10

Step1
Add all the numbers up. i.e. 25+20+15+12=70

Step 2
Count the number of data points. Here there are 4 numbers given i.e. 25,20,15,12

Step 3
We divide the sum of the numbers with the number of data points. So, 72/4=18.
18 is the Mean of 25,20,15,12

Practice Example's

1. Find the Mean of 15,12,3,10
2. Find the Mean of 8,12,5,7
3. Find the Average of 25,35,20,12
4. Find the Average of 18,22,17,13

HCF & LCM

Factors and Multiples: All the numbers that divide a number completely, i.e., without leaving any remainder, are called factors of that number.

For example,

24 is completely divisible by 1, 2, 3, 4, 6, 8, 12, 24. Each of these numbers is called a factor of 24 and 24 is called a multiple of each of these numbers.

How to find HCF orally:

HCF stands for Highest Common Factor, which is also known as GCD or Greatest Common Divisor. It is the highest number that divides two or more numbers exactly without leaving a remainder. HCF is an important concept in mathematics, and it is used in various applications like simplification of fractions, finding the smallest common denominator, etc.

There are three methods for finding HCF: Prime Factorization Method, Subtraction Method, and Division Method. Each of these methods is useful in different situations.

Prime Factorization Method:

This method involves finding the prime factors of the given numbers and then identifying the common prime factors. The product of these common prime factors gives us the HCF of the given numbers.

Example: Find the HCF of 24 and 36 using Prime Factorization Method.

Step 1: Find the prime factors of 24 and 36.

24 = 2 x 2 x 2 x 3

36 = 2 x 2 x 3 x 3

Step 2: Identify the common prime factors of 24 and 36.

2 x 2 x 3 = 12

Step 3: The product of common prime factors, 12, is the HCF of 24 and 36.

Subtraction Method:

This method involves repeatedly subtracting the smaller number from the larger number until both the numbers become equal. The last number obtained before the numbers become equal is the HCF of the given numbers.

Example: Find the HCF of 48 and 60 using Subtraction Method.

Step 1: Start by subtracting the smaller number from the larger number.

60 - 48 = 12

Step 2: Repeat the process by subtracting the smaller number (12) from the larger number (48).

48 - 12 = 36

Step 3: Again, subtract the smaller number (12) from the larger number (36).

36 - 12 = 24

Step 4: Continue the process until both numbers become equal.

24 - 12 = 12

12 - 12 = 0

Step 5: The last non-zero number obtained, 12, is the HCF of 48 and 60.

Division Method:

This method involves dividing the larger number by the smaller number and finding the remainder. Then divide the smaller number by the remainder and find the new remainder. Continue the process until the remainder is zero. The last non-zero remainder obtained is the HCF of the given numbers.

Example: Find the HCF of 72 and 90 using Division Method.

Step 1: Divide the larger number (90) by the smaller number (72) and find the remainder.

90 ÷ 72 = 1 with remainder 18

Step 2: Divide the smaller number (72) by the remainder (18) and find the new remainder.

72 ÷ 18 = 4 with remainder 0

Step 3: The last non-zero remainder obtained, 18, is the HCF of 72 and 90.

When to use each method:

Prime Factorization Method: This method is best used when the numbers are small and have small prime factors. It is also useful when we need to find the HCF of more than two numbers.

Subtraction Method: This method is best used when the difference between the given numbers is small. It is also useful when we need to find the HCF of two numbers.

Division Method: This method is best used when the numbers are large and have no common prime factors. It is also useful when we need to find the HCF of two numbers.

By using all three methods, we can verify our answers and get a better understanding of the concepts involved.

Exercise:

1. Find the HCF of 30 and 45 using prime factorization method.
2. Find the HCF of 18 and 24 using the subtraction method.
3. Find the HCF of 36 and 48 using the division method.
4. Find the HCF of 72, 90 and 126 using the prime factorization method.
5. Find the HCF of 120 and 150 using the subtraction method.

How to find LCM orally

LCM stands for Least Common Multiple, which is the smallest number that is a multiple of two or more given numbers. It is an important concept in mathematics and is used in various fields such as arithmetic, algebra, and number theory.

Methods of Identifying LCM:

There are two main methods to identify the LCM of given numbers: the prime factorization method and the addition method.

Prime Factorization Method:

In the prime factorization method, we find the prime factors of each given number and then multiply them together. To do this, we follow the steps below:

Step 1: Write the given numbers as a product of their prime factors. Step 2: Identify the common prime factors. Step 3: Multiply the common prime factors together. Step 4: Multiply any remaining prime factors together. Step 5: The product obtained in step 3 and step 4 is the LCM of the given numbers.

Example: Find the LCM of 12 and 18 using the prime factorization method.

Step 1: Find the prime factorization of each number.

12 = 2 x 2 x 3

18 = 2 x 3 x 3

Step 2: Write down the factors that appear in either of the two factorizations, using the highest exponent for each factor.

LCM = 2^2 x 3^2 = 12 x 9 = 36

Therefore, the LCM of 12 and 18 is 36.

Addition Method:

In the addition method, we find the LCM by adding the larger number repeatedly until we get a number that is divisible by all the given numbers. To do this, we follow the steps below:

Step 1: Write the given numbers horizontally. Step 2: Identify the largest number. Step 3: Check if the largest number is divisible by the other numbers. If it is, then it is the LCM. Step 4: If it is not divisible by the other numbers, add the largest number to itself. Step 5: Repeat step 4 until you get a number that is divisible by all the given numbers. Step 6: The final number obtained is the LCM of the given numbers.

Example: Find the LCM of 15 and 25 using the addition method.

Step 1: Write the numbers horizontally: 15, 25

Step 2: The largest number is 25.

Step 3: 25 is not divisible by 15.

Step 4: Add 25 to itself to get 50.

Step 5: 50 is not divisible by 15.

Step 6: Add 25 to 50 to get 75.

Step 7: 75 is divisible by both 15 and 25.

Therefore, the LCM of 15 and 25 is 75.

When to use each method:

The prime factorization method is best suited for finding the LCM of larger numbers as it is less time-consuming than the addition method. It is also useful when finding the LCM of more than two numbers.

On the other hand, the addition method is useful for finding the LCM of smaller numbers as it involves simple addition. It is also useful when only two numbers are given.

In general, it is a good practice to use both methods to check your answer and ensure its accuracy.

Practice

1. Find the LCM of 18 and 24 using the addition method.
2. Find the LCM of 15 and 25 using the addition method.
3. Find the LCM of 12 and 20 Using the Prime Factorisation Method
4. Find the LCM of 24 and 36 Using the Prime Factorisation Method
5. Find the LCM of 36 and 48 using prime factorization method.

Squaring Numbers

Squaring can be defined as 'multiplying a number by itself.'
There are many different ways of squaring numbers. Many of these techniques have their roots inmultiplication as squaring is simply a process of multiplication.

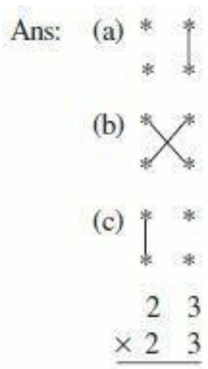
Examples: 3^2 is 3 multiplied by 3 which equals 9

The technique that we will study are: -

(A) SQUARING OF NUMBERS USING CRISSCROSS SYSTEM

The Urdhva-Tiryak Sutra (the Criss-Cross system) is by far the most popular system of squaring numbers amongst practitioners of Vedic Mathematics. The reason for its popularity is that it can be used for any type of numbers.

Ex: Find the square of



(a) First, we multiply 3 by 3 and get the answer as 9.(Answer at this stage is___________________9)

(b) Next, we cross multiply (2 × 3) and add it with (2 × 3).The final answer is 12. We write down 2 and carry over 1. (Answer at this stage is_29)

(c) Thirdly, we multiply (2 × 2) and add the 1 to it. The answer is 5.
The final answer is 529

Cubing Numbers

We know that squaring is multiplying a number by itself. Cubing can be defined as multiplying a number by itself and again by itself. Thus 3 × 3 × 3 is 27 and thus 27 is the cube of 3. When we cube a number we are said to have raised it to the power of 3. The cube of a number is expressed by putting a small three on the top right part of the number. For example,

$10^3 = 1000$
$11^3 = 1331$

Cubing is important while dealing with some algebraic equations and also while dealing with three-dimensional figures in geometry.

$2^3 = 2 \times 2 \times 2 = 8$
$3^3 = 3 \times 3 \times 3 = 27$
$4^3 = 4 \times 4 \times 4 = 64$
$5^3 = 5 \times 5 \times 5 = 125$
$6^3 = 6 \times 6 \times 6 = 216$
$7^3 = 7 \times 7 \times 7 = 343$
$8^3 = 8 \times 8 \times 8 = 512$
$9^3 = 9 \times 9 \times 9 = 729$
$10^3 = 10 \times 10 \times 10 = 1000$

Cube Roots of Perfect Cubes

The cube root is **often used to solve cubic equations**. In particular, it can be used to solve for the dimensions of a three-dimensional object of a certain volume

Unlike the schools methods of calculating Cube Roots, the technique used in Vedic Mathematics is very simple and fast!

By following the right steps, students can mentally calculate cube root of even big number such as 35937, 85184 and 493039 in no time at all.

Before we move ahead lets first see what is a square and cube of number

Lets take the example of the number 5

<u>When we "square" the Number 5:</u>
We Multiply the Number 5 by itself:

$5^2 = 5 \times 5 = 25$

<u>When we "cube" the number 5:</u>
We Multiply the 5 by itself 2 times:

$5^3 = 5 \times 5 \times 5 = 125$

Similarly, when we square the number 4, we do the following:

$4^2 = 4 \times 4 = 16$

And, when we cube the number 4, we

$4^3 = 4 \times 4 \times 4 = 64$

Let's now go ahead and see what is Cube-root of a number:
Cube-root is essentially the reverse of calculating the Cube of a Number

So, if 27 is the cube of 3, then 3 is the cube root of 27

Hence, finding Cube-root of a numbers is a technique to identify the number which has been multiplied by itself twice to obtain the cube.

Thus, if 8 is the cube of 2, then 2 is the cube-root of 8.

If 27 is the cube of 3 then 3 is the cube root of 27 and so on.

What's unique about vedic mathematics is that by following its techniques and steps we will be able to easily find the cube-roots of higher numbers such as 636056, 493039 and so on.

However, do note that this technique can only be used to find the cube root of Perfect cubes

Before we start calculating the Perfect cube-root of a Cube, we need to learn 3 Rules

1. **Rule 1: Memorize the cubes of numbers 1-15**

Key

Number/Cuberoot	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375

2. **<u>Rule 2: Identify and Memorise the relationship between digits of Cube-root and Cubes,</u>** for example

   a. if the cube of a number ends with 1, then its cuberoot also ends with 1
   So if the cube of a number is 133<u>1</u>, its cube root which is 1<u>1</u> also end with <u>1</u>

   b. If the cube of a number ends with 8, then its cube root will end with 2
   So if the cube of a number is 172<u>8</u>, its cube root which is 1<u>2</u> will end with <u>2</u>

The above relationship is constant for all Perfect cubes of Cube-roots

The below table is in live with the above principle

The last Digit of the Cube	The last digit of the Cube-root
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9
0	0

   c. As shown above cube roots ending with 1,4,5,6,9,0 correspond to their cubes, except for Cube-roots ending with 8 whose cube is 2(and vice versa) and Cube-roots ending with 3 whose cube is 7 (and vice versa)

3. **Rule 3: Whenever you are solving a cube to find out its cube-root, you must put a slash before the last 3 digits**

   a. For example

      i. To find the Cube-root of 12167, represent the same as follows:

Left hand Side

LHS **RHS**

12 | 167

Right hand Side

      ii. To find the Cube-root of 830584, represent the same as follows:

LHS **RHS**

830 | 584

      iii. Although it's not a rule, generally we prefer to solve RHS first

Now that we have learnt the 3 Rules, let's start solving

Example 1: 175616

1. **Step 1: We will be solving the cube-root in 2 parts**

	LHS	RHS
	175	616
Answer		

2. **Step 2: As per the Relationship Rule of Cube and Cuberoot:**
   a. Find RHS: We see that the cube 175616 end with 6, so the cube root of this number will also be 6, so RHS of our answer is 6

	LHS	RHS
	175	616
Answer		6

3. **Step 3: Using the Rule no 1 of Memorization of numbers from 1-10**

	LHS	RHS
	175	616
Answer		6

Key

Number/Cuberoot	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

a. Find LHS: To find Answer on LHS, we take the number on LHS which is 175

b. Now, we need to find two perfect cubes between which the number 175 lies in the number line.

From the key, we find that 175 lies between the perfect cubes
125(the cube of 5) and
216(the cube of 6).

c. Now, between the 2 cubes we take the cuberoot of the lower cube, which is 5

	LHS	RHS
	175	616
Answer	5	6

So, the Cube-root of 175616 is 56

Example 2: 830584

1. **Step 1: We will be solving the cube-root in 2 parts**

	LHS	RHS
	830	584
Answer		

2. **Step 2: As per the Relationship Rule of Cube and Cuberoot:**
   a. Find RHS: We see that the cube 830584 end with 4, so the cube root of this number will also be 4, so RHS of our answer is 4

	LHS	RHS
	830	584
Answer		4

3. **Step 3: Using the Rule no 1 of Memorization of numbers from 1-10**

	LHS	RHS
	830	584
Answer		4

Key

Number/Cuberoot	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

a. Find LHS: To find Answer on LHS, we take the number on LHS which is 830

b. Now, we need to find two perfect cubes between which the number 830 lies in the number line.

From the key, we find that 830 lies between the perfect cubes
729(the cube of 9) and
1000(the cube of 10).

c. Now, between the 2 cubes we take the cuberoot of the lower cube, which is 9

	LHS	RHS
	830	584
Answer	9	4

So, the Cube-root of 830584 is 94

Example 3: 1601613

1. **Step 1: We will be solving the cube-root in 2 parts**

	LHS	RHS
	1601	613
Answer		

2. **Step 2: As per the Relationship Rule of Cube and Cuberoot:**
   a. Find RHS: We see that the cube 1601613 ends with 3, so the cube root of this number will end with 7, so RHS of our answer is 7

	LHS	RHS
	1601	613
Answer		7

3. **Step 3: Using the Rule no 1 of Memorization of numbers from 1-15**

Key

Number/Cuberoot	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375

	LHS	RHS
	1601	613
Answer		7

a. Find LHS: To find Answer on LHS, we take the number on LHS which is 1601

b. Now, we need to find two perfect cubes between which the number 1601 lies in the number line.

From the key, we find that 1601 lies between the perfect cubes 1331(the cube of 11) and 1728(the cube of 12).

c. Now, between the 2 cubes we take the cube-root of the lower cube, which is 11

	LHS	RHS
	1601	613
Answer	11	7

So, the Cube-root of 1601613 is 117

LIVE MATH COMPETITIONS AND LEAGUE

Practice Sheet (Part A)

Find the Cube-root of the Following:

Exercise 1: 13824

1. **Step 1: We will be solving the cube-root in 2 parts**

	LHS	RHS
Answer	__	___ _

2. **Step 2: As per the Relationship Rule of Cube and Cuberoot:**
   a. Find RHS: We see that the cube 13824 ends with __, so the cube root of this number will end with _, so RHS of our answer is __

	LHS	RHS
Answer	__	___ _

3. **Step 3: Using the Rule no 1 of Memorization of numbers from 1-15**

Key

Number/Cuberoot	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375

	LHS	RHS
Answer	__	___ _

a. Find LHS: To find Answer on LHS, we take the number on LHS which is __

b. Now, we need to find two perfect cubes between which the number __ lies in the number line.

From the key, we find that __ lies between the perfect cubes ___(the cube of _) and ___(the cube of _).

c. Now, between the 2 cubes we take the cube-root of the lower cube, which is _

	LHS	RHS
Answer	__ _	___ _

So, the Cube-root of 13824 is __

Exercise 2: 97336

1. **Step 1: We will be solving the cube-root in 2 parts**

	LHS	RHS
Answer	__	___ _

2. **Step 2: As per the Relationship Rule of Cube and Cuberoot:**
   a. Find RHS: We see that the cube 97336 ends with __, so the cube root of this number will end with _, so RHS of our answer is __

	LHS	RHS
Answer	__	___ _

3. **Step 3: Using the Rule no 1 of Memorization of numbers from 1-15**

Key

Number/Cuberoot	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375

	LHS	RHS
Answer	__	___ _

a. Find LHS: To find Answer on LHS, we take the number on LHS which is __

b. Now, we need to find two perfect cubes between which the number __ lies in the number line.

From the key, we find that __ lies between the perfect cubes ___(the cube of _) and ____(the cube of _).

c. Now, between the 2 cubes we take the cube-root of the lower cube, which is _

	LHS	RHS
Answer	__ _	___ _

So, the Cube-root of 97336 is __

Exercise 3: 2299968

1. **Step 1: We will be solving the cube-root in 2 parts**

	LHS	RHS
Answer	___	____

2. **Step 2: As per the Relationship Rule of Cube and Cuberoot:**
   a. Find RHS: We see that the cube 2299968 ends with __, so the cube root of this number will end with _, so RHS of our answer is __

	LHS	RHS
Answer	___	____

3. **Step 3: Using the Rule no 1 of Memorization of numbers from 1-15**

Key

Number/Cuberoot	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375

	LHS	RHS
Answer	___	____

a. Find LHS: To find Answer on LHS, we take the number on LHS which is ___

b. Now, we need to find two perfect cubes between which the number ___ lies in the number line.

From the key, we find that ____ lies between the perfect cubes ___(the cube of __) and ___(the cube of __).

c. Now, between the 2 cubes we take the cube-root of the lower cube, which is _

	LHS	RHS
Answer	___	____

So, the Cube-root of 2299968 is __

Practice Sheet (Part B)

Find the Cube Root Mentally

1. **12167 = ______**
2. **29791 = ______**
3. **74088 = ______**
4. **262144 = ______**
5. **438976 = ______**
6. **970299 = ______**
7. **117649 = ______**
8. **704969 = ______**
9. **175616 = ______**
10. **1520875 = ______**

Perfect Square Roots

Square root of a number is a value, which on multiplication by itself gives the original number. The square root of a number is the inverse of squaring a number. Hence, squares and square roots are related concepts.

Unlike the school's methods of calculating Square Roots, the technique used in Vedic Mathematics is very simple and fast!

By following the right steps, students can mentally calculate square root of even big numbers such as 4761, 4624 in no time at all!

Before we move ahead lets first see what is a square of number.

Let's take the example of the number 5.

When we "square" the Number 5:
We Multiply the Number 5 by itself:

$5^2 = 5 \times 5 = 25$

Similarly, when we square the number 4, we do the following:

$4^2 = 4 \times 4 = 16$

Let's now go ahead and see what is Square-root of a number:
Square-root is essentially the reverse of calculating the Square of a Number

So, if 9 is the Square of 3, then 3 is the square root of 9.

Hence, finding Square-root of a number is a technique to identify the number which has been multiplied by itself to obtain the Square.

Thus, if 16 is the square of 4, then 4 is the square-root of 16.

If 25 is the square of 5 then 5 is the square root of 25 and so on.

But what's unique about Vedic mathematics is that by following its techniques and steps we will be able to easily find the square-roots of higher numbers such as 3136 and 9801 and so on.

However, do note that this technique can only be used to find the square root of Perfect squares

Before we start to calculate Perfect Square-root of a Square, we need to learn 3 Rules

1. **<u>Rule 1: Memorize the Squares of numbers 1-10</u>**

Number	Squares
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

2. **<u>Rule 2: Identify and Memorise the relationship between last digits of Square-root and Squares,</u>** for example

   Referring the Table **Key**, we will need to observe and memorise the relationship between the last digit of the Squares and the last digit of their square root.

   a. if the square of a number ends with 1, then its Square-Root will end with either 1 or 9 (because 1 x 1 = 1 and 9 x 9 is 81)

      - So, if the square of a number is 2601, we can immediately assume that its square root will either end with 1 or 9.
        In the case of 2601 the square root is 51.
      - Likewise, if the given square is 4761, its square root will either end with 1 or 9.
        In this case for 4761, its square root is 69

   b. Similar to the 1 and 9 relationship, if a number ends in 4 the square root ends in 2 or 8(because 2 × 2 is 4 and8× 8 is 64)
   c. If a number ends in 9, the square root ends in 3 or 7(because 3 × 3 is 9 and 7 × 7 is 49)
   d. If a number ends in 6, the square root ends in 4 or 6(because 4 × 4 is 16 and 6× 6 is 36)
   e. If the number ends in 5, the square root ends in 5(because 5 × 5 is 25)
   f. If the number ends in 0, the square root also ends in 0(because 10 × 10 is 100)

The above relationship is constant for all Perfect Squares of Square -roots

The below table is in line with the above principle

The last Digit of the Square	The last digit of the Square-root
1	1 or 9
4	2 or 8
9	3 or 7
6	4 or 6
5	5
0	0

Now, I want you to look at the column on the left. It reads 'Last digit of the square' and the numbers contained in the column are 1,4, 9, 6, 5and 0. Note that the numbers 2, 3, 7and 8 are absent in the column. That means there is no perfect square which ends with the numbers 2, 3, 7 or 8. Thus we can say:

'A perfect square will never end with the digits 2,3, 7 or 8'

Now we know how to find the last digit of a Square-Root based on the last digit of the Square, although most of the times there are 2 possibilities out of which only 1 is correct.

Secondly, we still need to find out the digits of the square root. So now let's proceed with examples so that we learn how to arrive at the Final Last Digit and the other digits of the square root.

3. **Rule 3: Before we proceed there is one final table that needs to be memorised, as this will help easily determine the Square roots.**

Number	Squares
10	100
20	400
30	900
40	1600
50	2500
60	3600
70	4900
80	6400
90	8100
100	10000

Now that we have learnt the **3** Rules, let's start solving

Example 1: 4624

1. Based on the Rule Number 2 relationship between last digits of Squares and Square roots:
    a. The number 4624 ends with 4. Hence, the square root ends with 2 or 8. The answer at this stage is 2 or 8.

Number	Squares
10	100
20	400
30	900
40	1600
50	2500
60	3600
70	4900
80	6400
90	8100
100	10000

2. Next we take in the complete number 4624, and using Table in Rule no 3, we can see that 4624 lies between 3600(which is the square of 60) and 4900(which is the square of 70).
    a. Hence the Square root also lies between 60 and 70
    b. Of all numbers between 60 and 70 (61,62,63,64,65,66,67,68,69), the only numbers ending with 2 and 8 are 62 and 68. However only one of them is correct, lets find the right one.
    c. Observe the squares 3600 and 4900

60 – 3600

4624

70 – 4900

    d. Is 4624 closer to the smaller number 3600 or closer to the bigger number 4900?
        i. If the number 4624 is close to 3600, then take smaller 62 as the square root
        ii. If the number 4624 is close to 4900, then take smaller 68 as the square root
        iii. However, in this case we can see 4624 is closer to 4900, the right answer is **68**

Example 2: 5776

1. Based on the Rule Number 2 relationship between last digits of Squares and Square roots:
   a. The number 5776 ends with 6. Hence, the square root ends with 4 or 6. The answer at this stage ends with 4 or 6.

Number	Squares
10	100
20	400
30	900
40	1600
50	2500
60	3600
70	4900
80	6400
90	8100
100	10000

2. Next we take in the complete number 5776, and using Table in Rule no 3, we can see that 5776 lies between 4900(which is the square of 70) and 6400(which is the square of 80).
   a. Hence the Square root also lies between 70 and 80
   b. Of all the numbers between 70 and 80 (71,72,73,74,75,76,77,78,79), the only numbers ending with 4 and 6 are 74 and 76. However only one of them is correct, lets find the right one.
   c. Observe the squares 4900 and 6400

      70 – 4900

      5776

      80 – 6400

   d. Is 5776 closer to the smaller number 4900 or closer to the bigger number 6400?
      i. If the number 5776 is close to 4900, then take smaller 74 as the square root
      ii. If the number 5776 is close to 6400, then take smaller 76 as the square root
      iii. However, in this case we can see 5776 is closer to 6400, the right answer is **76**

Example 3: 8649

1. Based on the Rule Number 2 relationship between last digits of Squares and Square roots:
    a. The number 8649 ends with 9. Hence, the square root ends with 3 or 7. The answer at this stage ends with 3 or 9.

Number	Squares
10	100
20	400
30	900
40	1600
50	2500
60	3600
70	4900
80	6400
90	8100
100	10000

2. Next we take in the complete number 8649, and using Table in Rule no 3, we can see that 8649 lies between 8100(which is the square of 90) and 10000(which is the square of 100).
    a. Hence the Square root also lies between 90 and 100
    b. Of all the numbers between 90 and 100 (91,92,93,94,95,96,97,98,99), the only numbers ending with 3 and 7 are 93 and 97. However only one of them is correct, lets find the right one.
    c. Observe the squares 8100 and 10000

        90 – 8100

        8649

        100 – 10000

    d. Is 8649 closer to the smaller number 8100 or closer to the bigger number 10000?
        i. If the number 8649 is close to 8100, then take smaller 93 as the square root
        ii. If the number 8649 is close to 10000, then take smaller 97 as the square root
        iii. However, in this case we can see 8649 is closer to 8100, the right answer is **93**

LIVE MATH COMPETITIONS AND LEAGUE

Practice Sheet (Part A)

Find the Square-root of the Following:

Example 1: 729

1. Based on the Rule Number 2 relationship between last digits of Squares and Square roots:
    a. The number 729 ends with ___. Hence, the square root ends with ___ or ___. The answer at this stage ends with ___ or ___.

Number	Squares
10	100
20	400
30	900
40	1600
50	2500
60	3600
70	4900
80	6400
90	8100
100	10000

2. Next we take in the complete number 729, and using Table in Rule no 3, we can see that 729 lies between ______(which is the square of ___) and _______(which is the square of ___).
    a. Hence the Square root also lies between ___ and ____
    b. Of all the numbers between ____ and ____ (___,___,___,___,___,___,___,___,___), the only numbers ending with ___ and ___ are ___ and ___. However only one of them is correct, lets find the right one.
    c. Observe the squares _____ and ______

        ___ – _____

        729

        ___ – ______

    d. Is _____ closer to the smaller number _____ or closer to the bigger number _______?
        i. If the number _____ is close to _____, then take smaller ___ as the square root
        ii. If the number _____ is close to ______, then take smaller ___ as the square root
        iii. However, in this case we can see 729 is closer to ______, the right answer is ____

Find the Square-root of the Following:

Example 2: 1849

1. Based on the Rule Number 2 relationship between last digits of Squares and Square roots:
    a. The number 1849 ends with __. Hence, the square root ends with __ or __. The answer at this stage ends with __ or __.

Number	Squares
10	100
20	400
30	900
40	1600
50	2500
60	3600
70	4900
80	6400
90	8100
100	10000

2. Next we take in the complete number 1849, and using Table in Rule no 3, we can see that 1849 lies between ______(which is the square of __) and _______(which is the square of __).
    a. Hence the Square root also lies between __ and ___
    b. Of all the numbers between ___ and ___ (__,__,__,__,__,__,__,__,__), the only numbers ending with __ and __ are __ and __. However only one of them is correct, lets find the right one.
    c. Observe the squares _____ and ______

        __ – _____

        1849

        __ – ______

    d. Is _____ closer to the smaller number _____ or closer to the bigger number _______?
        i. If the number _____ is close to _____, then take smaller __ as the square root
        ii. If the number _____ is close to ______, then take smaller __ as the square root
        iii. However, in this case we can see 1849 is closer to ______, the right answer is ____

LIVE MATH COMPETITIONS AND LEAGUE

Find the Square-root of the Following:

Example 2: 6084

1. Based on the Rule Number 2 relationship between last digits of Squares and Square roots:
    a. The number 6084 ends with __. Hence, the square root ends with __ or __. The answer at this stage ends with __ or __.

Number	Squares
10	100
20	400
30	900
40	1600
50	2500
60	3600
70	4900
80	6400
90	8100
100	10000

2. Next we take in the complete number 6084, and using Table in Rule no 3, we can see that 6084 lies between ______(which is the square of __) and _______(which is the square of __).
    a. Hence the Square root also lies between __ and ___
    b. Of all the numbers between ___ and ___ (__,__,__,__,__,__,__,__,__), the only numbers ending with __ and __ are __ and __. However only one of them is correct, lets find the right one.
    c. Observe the squares _____ and ______

       __ – _____

       6084

       __ – ______

    d. Is _____ closer to the smaller number _____ or closer to the bigger number _______?
        i. If the number _____ is close to _____, then take smaller __ as the square root
        ii. If the number _____ is close to ______, then take smaller __ as the square root
        iii. However, in this case we can see 6084 is closer to ______, the right answer is ____

Practice Sheet (Part B)

Find the Square Root Mentally

1. 6375 = ______

2. 6400 = ______

3. 7396 = ______

4. 8281 = ______

5. 6724 = ______

6. 2809 = ______

7. 1936 = ______

8. 3721 = ______

9. 784 = ______

10. 3136 = ______

References

Maharaja Jagadguru Swami Sri Bharati Krishna Tirthaji.

- Vedic Mathematics: Sixteen Simple Mathematical Formulae from the vedas. Motilal Bansridas Publishers, 1965.

Bill Hanley

- Speed Mathematics

Ann Cutler and Rudolph Mc Shane

- The Trachtenberg Speed System of Basic Arithmetic

James Glover

- Vedic mathematics for schools (Part 1,2 and 3)

Arthur Benjamin and Michael Shermer

- Secrets of Mental Math

Robert Fountain and Jan van Koningsveld

- The Mental Calculators Handbook

Mr.Gaurav Tekriwal

- Maths Sutras- The Art of Vedic Speed Calculation. Penguin Random House India,2015
- Maths Sutras- From around the World.

Mr Dhaval Bhatia

- Vedic Mathematics Made Easy